

WHY WAS ALCHOURRÓN AFRAID OF SNAKES?

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Abstract

In the last papers published by Alchourrón, he attacked non-monotonic logics, which he considered philosophically unsound for the representation of defeasible reasoning. Instead of a non-monotonic consequence relation, he proposed a formal representation of defeasibility based on an AGM-like revision of implicit assumptions connected to the premises. Given that this is a procedure to generate non-monotonic logics, it is not clear, from a mathematical standpoint, why he was so suspicious of such logics. In the present paper we try to answer this question based on Alchourrón's convictions about epistemology, particularly the epistemology of law. We also propose another revision operator on theories called refinement, which provides a faithful representation of the sort of epistemic change considered by Alchourrón as intrinsic to defeasibility.

KEY WORDS: defeasibility - non-monotonic logics - belief revision - deontic logic.

Resumen

En sus últimos escritos publicados, Alchourrón desarrolló un ataque contra las lógicas no-monotónicas, que consideraba filosóficamente inadecuadas a la representación del razonamiento derrotable. En vez de una relación de consecuencia no-monotónica, él propuso una representación formal de la derrotabilidad basada en un modelo de revisión (tipo-AGM) de las asunciones implícitas ligadas a las premisas. Dado que éste es un procedimiento para generar lógicas no-monotónicas, no está claro –en el nivel matemático– por qué él era tan desconfiado de tales lógicas. En el presente texto intentamos contestar a esta pregunta basados en las convicciones de Alchourrón sobre epistemología, particularmente la epistemología jurídica. También proponemos otro operador de revisión aplicado a las teorías, llamado refinamiento, que proporciona una representación fiel de la especie de cambio epistémico considerado por Alchourrón como intrínseco a la derrotabilidad.

PALABRAS CLAVE: derrotabilidad - lógicas no-monotónicas - revisión de creencias - lógica deóntica.

1. Introduction

Alchourrón was one of the leading figures in the development of deontic logic during the second half of the last century. He made decisive contributions to the clarification of key concepts and also to the development of logical tools to represent the normative discourse. Among them,

two deserve a special mention in the present paper: (i) the study of the logical properties of norms as opposed to the logic of normative propositions (Alchourrón, 1969), the latter understood as a basis for a theory of normative systems, fully developed in (Alchourrón and Bulygin, 1971) together with his co-author Eugenio Bulygin; and (ii) the study of the logical properties of derogation of norms and revision of normative systems, together with the logician David Makinson (beginning with (Alchourrón and Makinson, 1981) and culminating with the so called AGM model, (Alchourrón *et al.*, 1985)), which later became the foundation of a new field of research, namely, belief revision, with a relevant place also in artificial intelligence (knowledge representation) and contemporary studies in epistemology.

As a Law School undergraduate student, I was anxious to read everything and learn from Alchourrón about logic and about legal theory. His philosophical insights, clarity of exposition and precision of analysis were not only an inspiration, but the very reason of my interest in the study of law. The sad news of his premature death, in 1996, represented to me the loss of a hero and I sincerely regret that I never had the opportunity to know him personally.

In the 1990's, deontic logic was living the heyday of non-monotonic logics, which very soon dominated the mainstream, if not of deontic logic itself, certainly of artificial intelligence and law. Such logics are very appealing to represent *defeasibility* in legal and moral reasoning, i.e. the fact that in legal and moral reasoning some obligations are dropped in presence of new circumstances (*prima facie* duties). For instance, it is normally forbidden to kill, but such action may be permitted if the killer acted in self defense. They also provide a nice representation of moral dilemmas and normative conflicts where two obligations coexist but may become inconsistent in some particular circumstance. For instance, a doctor who is obliged to tell the truth to his patient about his condition and also obliged to apply his best efforts to cure in a situation where he believes the truth will impair the patient's recovery.

Such examples are not easily dealt with within classical logic where monotonicity holds. If the prohibition to kill is derived using the classical (deontic) consequence relation, it will continue to be so whether the killer acted in self defense, was smoking, wore glasses, etc. If there is an obligation to tell the truth about the disease, this obligation is derived whether the truth will hurt, the hospital is green, or whatever new circumstance or premise is considered. But the same holds for the obligation to make the best for the patient. In a situation where the truth may compromise recovery, the doctor is committed to both tell the truth and

not tell the truth. The problem is that the deontic version of the principle of contradiction, $\neg(Oa \wedge O\neg a)$,¹ leads to the trivialization of the system of norms. Hence, monotonicity and the deontic principle of non-contradiction seem to be obstacles to the representation of moral dilemmas.

The characteristic feature of non-monotonic logics is that the consequence relation (usually represented by the snake \vdash) fails monotonicity, that is, the principle “if $A \vdash x$ and $A \subseteq B$ then $B \vdash x$ ” is not valid. This means, contrary to what happens in classical logic, that some conclusions may be lost with the introduction of new premises, which seems exactly what is demanded for the representation of *prima facie* duties and deontic dilemmas. However, Alchourrón thought otherwise:

“...there is no need for a logic of defeasible norms (norms of *prima facie* obligations and permissions) because behind the requirement for such logics, as well as behind the requirement for non-monotonic logics, now in fashion in artificial intelligence, lies a mixture of a standard notion of consequence (or conditional) and the change of our (normative) premises in a dynamic perspective.” (Alchourrón, 1993, p.44)

According to Alchourrón, those who advance that defeasibility demands a new consequence relation make confusion with the classical derivation based on a previous revision of the premises. This claim was acknowledged in the deontic logic community in a paper published in 1993 and was the object of criticism and bewilderment. Loui considered Alchourrón’s attack a “*consequence of his fancy for revision*” (Loui, 1997, p. 348), the model he created in co-authorship with David Makinson. In legal philosophy, Peczenik (1996) upheld the insufficiency of the AGM model to give an account of the weighing and balancing of reasons which are, according to him, characteristic of legal reasoning.

Recently Makinson published a book on defeasible logics, where different ways of generating non-monotonic logics out of classical logic are studied (Makinson, 2005). We read in its preface:

“The book is dedicated to the memory of my former co-author Carlos Alchourrón, who died in January 1996. Despite his fascination with the logic of belief revision he distrusted its neighbor non-monotonic logic, and I could never convince him otherwise. While writing this book, I realized that it is a continuation of our conversations”.

¹ “ \neg ” is the symbol for classical negation and “ \wedge ” the symbol for classical conjunction.

Makinson's point seems to be that such a move (first revise, then derive) is a way of generating a non-monotonic consequence relation and Alchourrón himself worked on a defeasible conditional, which is a snake in the object-language of the logic.

So why was he so suspicious of snakes?

It is the purpose of this paper to answer this question. Alchourrón was certainly *not* claiming that among different constructions of non-monotonic consequence relations, the one involving revision of the premises should be chosen. His main concern was the philosophical darkness of any non-monotonic *consequence relation*, deontic or not. To him, the snake *conceals* a change in epistemic state and this is poison!

Although this is a general claim for the representation of inferences in any domain of knowledge I will concentrate the discussion on his convictions about legal theory and the role of legal dogmatics in the description and systematization of norms. Alchourrón's attack may be justified if we depart from the mathematical level and talk about adequacy of representation with relation to his convictions about epistemology, particularly the epistemology of law.

As it will be clear, Alchourrón's defeasible conditionals were developed in order to show the bridge between defeasibility and revision. But if we are embedded in Alchourrón's concerns, we may become suspicious even of his defeasible conditionals. I propose an alternative revision operator, which I call *refinement*, to represent the operation of epistemic change favored by Alchourrón as the antidote to the concealing snakes.

The paper is organized as follows. In Section 2, I present very briefly two different ways (the one syntactic, the other semantic) of obtaining defeasible logics and compare them with Alchourrón's logic of contributory conditionals. Then, in Section 3, I explore Alchourrón's representation of normative systems based on his logic of normative propositions, whereby his convictions on the role of (legal) science will become clearer. In section 4, I suggest the model of refinement to suit such epistemic moves in the dogmatic reconstruction of normative systems. In section 5, I conclude with some final considerations.

2. Was not Alchourrón himself snaking?

2.1 *Visiting the nest*

Non-monotonic logics were developed in artificial intelligence to represent inferences under uncertainty. The underlying idea is that an agent who has not enough information infers based on presumptions

about how the world normally is. There are several ways to build a snake, adding implicit assumptions to the (classical) consequence relation and making them vary with consistency constraints. In face of new premises (information) the set of implicit assumptions may be reduced in order to preserve consistency and this is why some conclusions are lost. Makinson distinguishes three basic routes from classical to non-monotonic logics (Makinson, 2005): (i) *default assumptions*: add a set of sentences to the basic stock of premises with consistency constraints; (ii) *default valuations*: restrict the set of valuations of the model in such a way that the restriction varies if new premises are considered; (iii) *default rules*: insert additional inference rules, with consistency checks on its application.

For the purposes of this paper it is sufficient to discuss default assumptions and default valuations as well as defeasible conditionals, which are very close neighbors. So let us pay a very short visit to this nest of snakes.

Classical logic consists of a structure $S=\{A,R\}$ where A is a set of formulas (axioms) and R a set of rules of inference, from which a consequence relation \vdash between a set of formulas and a formula of the language and a consequence operator Cn on sets of formulas are defined in the usual way. A default-assumption logic may be defined within a structure $S_K=\{A,K,R\}$ where K is a set of formulas representing the basic assumptions. A *default-assumption consequence relation* \vdash_K (alias C_K) is defined as:

$A \vdash_K x$ if and only if $A, K' \vdash x$ for all subsets $K' \subseteq K$ which are maximally consistent with A (notation $K' \in (max(K,A))$)²

or using the consequence operator:

$$C_K(A) = \cup\{Cn(A \cap K') : K' \in (max(K,A))\}.$$

A less radical approach bases the derivation on a selection s of maximal subsets of K which are consistent with A, i.e. $\cup\{Cn(A \cap K') : K' \in s(max(K,A))\}$. To see defeasibility, take $K=\{p \rightarrow \neg f, b \rightarrow f\}$ ³ and notice that $\{b\} \vdash_K f$ but it may be the case, for instance if $s(max(K, \{b,p\}))=max(K, \{b,p\})$, that $\{b,p\} \vdash_K \neg f$. We may read the example as “a bird (normally) flies” but “if the bird is a penguin, then it is not the case that it (normally) flies”.

² $K' \in max(K,A)$ if and only if:

- (i) $K' \subseteq K$
- (ii) $K' \cap A$ is consistent
- (iii) there is no K'' , such that $K' \subset K''$ satisfying (i) and (ii)

A second route to defeasibility is semantic. For convenience, in the present paper we will work with possible worlds in a model $M = \{V, [.] \}$ where V is a set of classical valuations (worlds) and $[.]$ is an interpretation function taking us from each sentence to the set of valuations in which the sentence holds (or is true). The function $[.]$ satisfies the well known classical truth conditions. We call $[a]$ the proposition expressed by sentence a . By $[A]$ we mean the set $\cup\{[a]; a \in A\}$. We say that x is a logical consequence of A (notation $A \models x$) if and only if $[A] \subseteq [x]$. To get a defeasible consequence relation we need to restrict the set of worlds in which the premises are evaluated and make them vary with changes in the set of explicit premises. In order to do that we introduce a binary preference relation " $<$ " on the set of worlds within the model $M = \{V, [.] , < \}$. We say that w is a minimal element of a set of worlds W if and only if there is no $v \in W$, such that $v < w$. By $\min_{<} [A]$ we understand the set of minimal elements of proposition $[A]$ under the assumed preference relation. A non-monotonic consequence relation is then obtained by the following definition:

$A \approx_{<} x$ if and only if $\min_{<} [A] \subseteq [x]$

To check defeasibility, take a model such that $V = \{w, v\}$, $[a] = \{w, v\}$, $[b] = \{w\}$, $[c] = \{v\}$ and $< = \{(w, v)\}$. We have that $\min_{<} [a] \subseteq [b]$ and so $\{a\} \approx_{<} b$, but $\min_{<} [\{a, c\}] \not\subseteq [b]$, which means that $\{a, c\} \not\models b$.

Such non-monotonic consequence relation corresponds to Bengt Hansson's conditional obligations in his dyadic deontic logic (Hansson, 1969). Hansson's goal was to give a proper account of contrary-to-duty obligations and he interpreted the minimal elements in this semantics as the "*best worlds*". If some circumstance is given, interpreted as a violation of an ideal obligation, you ought to do what is true in all remaining best worlds (make the best out of sad circumstances). He interpreted the relation $\{a\} \approx_{<} b$ as a conditional obligation connective in the object-language $O(b/a)$ read as " b is obligatory under circumstance a ".

If we index the preference relation to each world in which we are evaluating the sentences we get Lewis's counterfactual conditionals. The minimal elements of each relation $<_w$ are here interpreted as the set of worlds w' which are most similar to w . By $\min_w [a]$ we mean the set of minimal elements of $[a]$ under the relation " $<_w$ ". Now, a counterfactual conditional $a > b$ is true at a world w if and only if $\min_w [a] \subseteq [b]$. The intuitive reading is that if a were true in the (actual) world w then b would also be true, since it is true in all most similar worlds in relation to w .

Each similarity relation provides, for each world and a proposition, a subset of worlds (the minimal elements) of this proposition. Hence

it is clear that the family of similarity relations in Lewis's model may be replaced by a choice function Ch having a world and a proposition as argument and a subset of worlds as value. Then we have that $a > b$ is true at a world w if and only if $Ch([a], w) \subseteq [b]$. We may map both approaches if we stipulate for every sentence a and world w that $Ch([a], w) = \min_w [a]$. Stipulating suitable constraints on the choice function or corresponding restrictions on the similarity relation we get the well known Lewis's logics of counterfactual conditionals V, VN and VT (Lewis, 1973).

We may specify that the choice function always applies to a single (the actual) world or we may stipulate a constraint on Ch such that $Ch([a], w) = Ch([a], w')$ for all $w, w' \in V$. If we do that there is no need for a second argument (worlds) in the choice function. Lewis calls the systems corresponding to this last constraint Absolute, getting the counterparts VA, VNA and VTA of his counterfactual logics (Lewis, 1973). These last systems can be built using Åqvist construction by means of a monadic selection operator f in the object language which corresponds to the choice function on propositions in the model (Åqvist, 1973). The counterfactual conditional is then obtained by the definitional abbreviation $a > b = \Box(fa \rightarrow b)$ where \Box is a normal modal necessity operator.

These counterfactual conditionals may be interpreted as deontic conditionals in Hansson's fashion (Lewis, 1973, pp. 96-104). All these conditionals are non-monotonic. Their semantic provide a distinguished set of formulas which are theorems of a non-monotonic logic. Note that restricting the set of valuations has the same effect as adding premises to the antecedent: restricting the set of worlds we augment the set of propositions which are true in the selected worlds. Therefore, there is a visible translation of the default assumptions approach into the default valuations: $Ch[a] = [\cap\{a\}, \cup s(\max(K, \{a\}))]$.

In order to criticize the snakes, Alchourrón built his logic of contributory conditionals in the very neighborhood of this nest. Let us now visit Alchourrón.

2.2 Snakes do conceal and this is poison!

2.2.1 Precedents

Although Alchourrón's attack became widely known through his paper (Alchourrón, 1993), his uneasiness with non-monotonic consequence relations had already been manifested in two previous works (Alchourrón 1988a and 1988b). The first was presented in a conference held in Mia-

mi and the second was published in Italian. These works were provoked by his reading of Hilpinen's paper on normative conflicts (Hilpinen, 1987). The problem raised by Hilpinen is the existence of norms which do not properly conflict but may generate normative contradictions in particular circumstances. For instance the norms:

- N1. Those who commit murder ought to be punished
 N2. Minors ought not to be punished

According to Hilpinen these norms may coexist as binding in a normative system. However, in the particular circumstance where a minor commits murder the judge is both obliged to punish and not to punish the subject. Hilpinen's point is that the conflict is restricted to this special case and therefore none of the norms need to be deleted since they continue to provide consistent solutions to "normal" cases. Hilpinen makes a distinction between normative conflict and normative contradiction applied to conditional norms and puts forward that in an adequate deontic logic the principle $\neg O(a \wedge \neg a)$ should be valid but not the principle $\neg(Oa \wedge O\neg a)$, which amounts to the invalidation⁴ of the principle of deontic conjunction $(Oa \wedge Ob) \rightarrow (Oa \wedge b)$.

Alchourrón disagreed with Hilpinen. According to him, the distinction between normative conflicts and normative inconsistency just reveals that a set of norms may be inconsistent given a (possibly empty) set of facts. He saw no reason why we should call the empty case "contradiction" and the non-empty case "conflict" and reacted to the idea that in the former the system does not need to be revised (Alchourrón, 1988a).⁵ In his logic of normative propositions (deontic formulas are interpreted as propositions describing the existence of norms in a system) the principles $\neg(Oa \wedge O\neg a)$ and $\neg O(a \wedge \neg a)$ are both invalid given that there may be normative orders containing inconsistent norms. The case where a minor commits murder may only be consistently solved if we are based on a different normative system where the antecedent conditions of N1 and N2 are modified.

³ The arrow " \rightarrow " represents classical material implication.

⁴ In any normal modal logic.

⁵ *"Supongamos que α [el sistema de normas] es un conjunto inconsistente vía el conjunto de hechos F y que los enunciados descriptivos de F son verdaderos. ¿Puede α llevar a cabo su función práctica de servir de guía para el comportamiento? Creo que la respuesta a esta pregunta debe ser negativa, porque el sistema ha sobredeterminado el comportamiento humano, puesto que son exigidas todas sus acciones y sus omisiones"* (Alchourrón 1988a, p. 301).

This is already a step towards his later attack. Two points are relevant: first, he did not accept that a system could consistently solve both a general case and another more specific one if it assigns different and conflicting solutions to each of them; second, he did not accept a weakening of the logic to account of such inconsistency, emphasizing that its solution is a matter of decision, not logic.⁶

In that same year Alchourrón approached directly the problem of defeasibility (Alchourrón, 1988b). The starting point was David Ross's concept of *prima facie* duties. He recognized that such a concept is incompatible with the principle of *deontic strengthening the antecedent* ($(b \rightarrow Oa) \rightarrow (b \wedge c \rightarrow Oa)$) given that new facts may cancel existing duties, but restricted the analysis to situations where such facts are *explicitly* mentioned as conditions for the application of another incompatible norm within the system. Then he rejected the adequacy of Lewis's counterfactual conditionals given that its logic satisfies *modus ponens*, which, according to his analysis, should fail in any proper account of defeasibility. His argument is the following:

If one accepts *modus ponens* $(b \supset Oa) \rightarrow (b \rightarrow Oa)$, then, given that the consequent of such formula classically implies $(b \wedge c \rightarrow Oa)$, one also accepts $(b \supset SOa) \rightarrow (b \wedge c \rightarrow Oa)$, and thus admits that no fact *c* can cancel the obligation contained in the conditional *prima facie* obligation of doing *a* in circumstance *b*.

Of course that on the formal level it is perfectly possible to have a consistent system failing strengthening the antecedent but satisfying *modus ponens*, as in Lewis's systems VW and VC (Lewis, 1973). Alchourrón's argument actually advances that it is philosophically or intuitively incoherent to abandon the first principle while holding the latter.

The addition of *modus ponens* to Lewis's system V, getting VW, is semantically justified on the assumption of truth of the antecedent of a counterfactual in a particular world where the counterfactual holds. In this case the counterfactual has the truth-value of its consequent, i.e. it becomes a material conditional. But Alchourrón found no intuitive support for such semantic assumption, given that it brings about counter-intuitive examples (Alchourrón, 1995). Indeed, he believed that a counterfactual should not even be evaluated as true or false if its antecedent holds (in this case it is not properly a *counterfactual*).

Then he turned to Hansson's dyadic deontic logic in which neither deontic strengthening the antecedent nor deontic *modus ponens* hold and

⁶ This is a conviction Alchourrón inherited from von Wright (von Wright, 1982, p. 12).

considered that the cost of Hansson's logic is too high in terms of inferential power.⁷ Alchourrón pointed out that in Hansson's system the obligation to punish murderer adults cannot be derived from the set N1-N2, which would be undesirable.

The moral is simply that you shouldn't lose strengthening the antecedent (which corresponds to monotonicity in the meta-language of consequence relations). If you do, then you cannot coherently hold the validity of *modus ponens*, and this was too much for Alchourrón. Indeed, Alchourrón first believed that a logic failing *modus ponens* would be *misnamed*. At that point it seemed that Alchourrón was rejecting the very possibility of an adequate formal representation of defeasibility. However, such position was later reviewed after Makinson's contribution, particularly due to his influential paper (Makinson, 1993):

“Although I held this view strongly [that every conditional construction should satisfy *modus ponens*], I am now convinced (after Makinson's observation) that there are ordinary language conditional constructions which do not satisfy *modus ponens*, viz. defeasible conditionals, because they do not assert sufficient but contributory conditions”. (Alchourrón, 1995, p.98)

The notion of a contributory condition was later formally developed by Alchourrón, as we shall see in the next section. But its seeds were already present in these previous approaches. According to Alchourrón, intuitively, the reason we are inclined to believe that murders committed by minors ought not to be punished is that: i) we identify a conflict of obligations in this case; and ii) we solve the conflict by giving precedence to N2 over N1.⁸ Defeasibility happens in this dynamic of revision and such dynamic must be explicit in its formal representation as a condition of philosophical adequacy. The question is how such dynamic should be represented in a formal account of defeasibility.

⁷ “Sin embargo, el defecto principal del sistema [Hansson's dyadic deontic logic] está en su pérdida de fuerza inferencial. Esto es así porque junto con las consecuencias indeseables desaparecen algunas conclusiones muy obvias; hecho que descalifica a todo el sistema lógico” (Alchourrón, 1988b: 274).

⁸ “La argumentación intuitiva muestra que en este caso dos aspectos relevantes han sido tomados en cuenta: a) que la presentación formal muestra que hay un conflicto de obligaciones en el caso ($p \& r$), y b) que este conflicto se resuelve porque en este caso la norma (4) tiene prioridad sobre la norma (3). El que no es capaz de entender esto, no ha entendido el contenido conceptual del sistema A.” (Alchourrón 1988b, pp. 276-277).

Hence some relevant beliefs about defeasibility became entrenched in Alchourrón's thought: *first*, defeasibility of obligations is a matter of conflicting duties; *second*, in an intuitive level these conflicts are implicitly solved through revision of the normative premises; *third*, the formal representation of defeasibility should make explicit the dynamics of revision with an underlying classical logic, or at least a monotonic logic. Alchourrón's temper would not have been satisfied by limiting himself to the diagnosis and so he faced the challenge: the result of the enterprise appeared five years later.

2.2.2 Contributory conditionals

In a series of three articles (Alchourrón, 1993, 1995, 1996a) Alchourrón brought about a representation of non-monotonicity satisfying his philosophical concerns. He developed a general theory of defeasible conditionals, which can be combined with deontic modalities. I begin by presenting the general theory and let the deontic version to be discussed in the next section.

Defeasibility was represented in the object language through a conditional. Such conditional should formalize the intuitive notion of a *contributory condition*, i.e. the explicit antecedent is a necessary condition of a sufficient condition for the conclusion. This is so because the antecedent only detaches the consequent in the presence of implicit assumptions, that is, the antecedent together with its implicit assumptions is sufficient to the conclusion.

Hence the notion of a "sufficient condition" is the starting point of his construction. He uses S5 strict implication " \Rightarrow " ($a \Rightarrow b = \Box(a \rightarrow b)$ where \Box is S5 necessity operator) given that it translates the notion of logical consequence into the object language of his logic. Semantically we have that the strict conditional $a \Rightarrow b$ is true at a world in a model if and only if $[a] \subseteq [b]$. A defeasible conditional " $>$ " is defined in the object language as in (Åqvist, 1973), i.e. a strict conditional with implicit assumptions in the antecedent ($a > b = f a \Rightarrow b$). Semantically, the choice function Ch provides the true condition of the defeasible conditional ($a > b$ is true iff $Ch[a] \subseteq [b]$). The novelty with respect to Åqvist's construction is Alchourrón's interpretation of the selection f as a revision operator ($f a$ represents the joint assertion of a with its consistent implicit assumptions). He calls this logic DFT, which he proves to be equivalent to Lewis's VTA.

Given that the set of assumptions must be consistent with the explicit contributory condition, we may interpret $f a$ as a revision of a fixed

set K of implicit assumptions by sentence a , i.e. $fa = {}^*_K(a) = K^*a$ satisfying the following f -axioms which correspond to AGM revision postulates:⁹

Postulate	Selection operator f	Revision operator *_K
self deductibility	f1. $fa \rightarrow a$	$a \in K^*a$
equivalence	f2. $(a \leftrightarrow b) \rightarrow (fa \leftrightarrow fb)$	if $a \leftrightarrow b \in Cn(\mathcal{O})$ then $K^*a = K^*b$
consistency	f3. $\diamond a \rightarrow \diamond fa$	if $\neg a \notin Cn(\mathcal{O})$ then $K^*a \neq L$
hierarchical ordering	f4. $(f(a \vee b) \leftrightarrow fa) \vee (f(a \vee b) \leftrightarrow fb) \vee (f(a \vee b) \leftrightarrow (fa \vee fb))$	$(K^*(a \vee b) = K^*a) \vee (K^*(a \vee b) = K^*b) \vee (K^*(a \vee b) = K^*a \cup K^*b)$

These axioms are semantically valid given obvious restrictions on the function Ch , which consists in a form of generating non-monotonic logics, as we have seen above. Considering that “ \Rightarrow ” translates the relation of the (classical) logical consequence “ \vdash ” into the object language we have:

$$\vdash a \S b \text{ if and only if } K^*a \vdash b$$

The operation of revision K^*a is defined constructively through a selection of all maximal subsets of K which are consistent with a . First we define the operation of contraction $K \div a = \cup s(\max(K, \{\neg a\}))$, and then we use the so called Levi identity to define revision $K^*a = Cn(\{a\} \cap K \div \neg a)$. Given that K is a theory and it holds that $Cn(\{a\} \cap \cup s(\max(K, \{a\}))) = \cup (Cn(\{a\} \cap s \max(K, \{a\}))$ it follows that:

$$K^*a \vdash b \text{ if and only if } a \vdash_{\sim_K} b \text{ iff } b \in C_K(a)$$

which means that Alchourrón was snaking in the neighborhood if not doing non-monotonic logic (both syntactically and semantically).

But the point is that neither is he endorsing this latter step, nor is he endorsing the use of contributory conditionals of the form $a > b$. Does this make sense given that this latter step follows necessarily from the definitions? It does. His efforts consisted exactly in showing the logical equivalence

⁹ The reader who is familiar with belief revision may be missing the postulates of closure ($K^*a = Cn(K^*a)$), inclusion ($K^*a \subseteq Cn(K \cap \{a\})$) and vacuity (if $\neg a \notin K$ then $Cn(K \cap \{a\}) \subseteq K^*a$). To account of the postulate of closure we may interpret fa as a finite axiomatization of the theory $K^*a = Cn(K^*a)$. Postulates of inclusion and vacuity tells us that the simple expansion of a theory by a new proposition is a limiting case of revision. These postulates are absent in the axiomatization of the selection operator f . As Becher et al. show, the absence of these postulates permits Alchourrón’s contributory conditions to avoid Gärdenfors impossibility theorem (see Becher et al., 1999).

lence of the dynamics of revision with non-monotonic logics, so that he could conclude, in the logical level: *we do not need snakes*. To illustrate this point he compared two procedures to analyze the familiar Birds-Penguins example, one by general conditionals, and the other by defeasible conditionals:

Birds (b) fly (a)
 Penguins (p) do not fly ($\neg a$)

General (revised) Conditionals	Defeasible Conditionals
G1. $(b \wedge \neg p) \Rightarrow a$	d1. $b > a$
G2. $p \Rightarrow \neg a$	d2. $p > \neg a$

Both procedures avoid the inconsistent and counterintuitive conclusion that penguins, which are birds, fly and do not fly. Given a defeasible conditional where d2 is preferred to d1 we derive the correct conclusion that penguins do not fly. But in Alchourrón's formalization of defeasible conditionals d1 is $fb \Rightarrow a$ and d2 is $fp \Rightarrow \neg a$. Provided that the choice related to fb and fp is given by the equations $Ch[b] = [b \wedge \neg p]$ and $Ch[p] = [p]$ (so we have $fb \Leftrightarrow (b \wedge \neg p)$ and $fp \Leftrightarrow p$) the procedure of representation with defeasible conditionals is logically equivalent to the procedure that uses general (revised) conditionals.

As we have seen, Alchourrón abandoned his view that any inference relation or any conditional construction deserving the name should satisfy detachment. But the demand for *modus ponens* was still strong at the back of his mind, as we see in the following passages:

"I think that we have a natural and spontaneous tendency to understand ordinary language constructions as a way of holding that the antecedent is a kind of sufficient condition of the consequent, the weakest form of which would be material implication. This is equivalent to assuming that conditionals always allow us to detach the consequent in the presence of the antecedent" (Alchourrón, 1995, p. 98)

"I see no intuitive reason to have a conditional with modus ponens but without strengthening the antecedent, even though I understand the strong psychological tendency to require the fulfillment of modus ponens for every conditional construction" (Alchourrón, 1995, p. 101)

In his construction of contributory conditions, where implicit assumptions related to the antecedent are made explicit and revised in presence of inconsistencies, he recovers the notion of a sufficient condi-

tion in the conditional construction: it does not hold but we do not lose track of it. By this move, Alchourrón seems to have conciliated his understanding of ordinary language defeasible conditionals with his psychological adherence to *modus ponens*.

But note that Alchourrón was not saying that the sentences “birds normally fly” and “birds which are not penguins fly” are logically equivalent. As it should be clear, he was perfectly aware that defeasibility has to do with ampliative reasoning. His model of revision is ampliative and defeasible in its dynamics. The procedures (not the sentences) are equivalent, i.e. “*derivation from revisable premises*” is the same as “*derivation by default*”.

This is what he requires to advance his philosophical argument about adequacy: snakes are dangerous *because they conceal* the dynamic of epistemic change which is in the very nature of defeasibility.¹⁰ So the philosophical question Alchourrón is proposing is: *to conceal or not conceal?*

In the next section we turn our attention to Alchourrón’s arguments for *not concealing* concentrated in the domain of legal theory.

3. The science of law and the description of normative systems

As positivists, Alchourrón and his co-author Eugenio Bulygin are concerned with the scientific grounds of legal dogmatics. One of their main tenets is to make a sharp distinction between knowledge of what law is (description and systematization of the content of valid general norms) and its critical evaluation according to moral or political standards.

They propose an abstract model of such descriptive activity inspired in the scientific ideal of an axiomatic system. Under such approach, knowledge of law consists in the knowledge of the logical consequences of some basic propositions about the content of existent norms (normative propositions). Individual legal decisions are justified to the extent in which their content is a deduction from the described general norms.

Therefore general norms described by legal dogmatics may be taken as explanations (or justifications) of individual decisions, analogously to Hempel’s theory of explanation where particular facts are explained whenever deduced from general conditionals together with the description of events.

Just as the ideal pursued by empirical sciences is a consistent and complete explanation of natural phenomena (universal causation), the ide-

¹⁰ Alchourrón remarks that when he discusses the procedure which uses defeasible conditionals “*the nature of the total enterprise, which involves a revision of our premises, is concealed, and for this reason might be dangerously misleading*”. (Alchourrón, 1993, p. 78)

al of legal science is the description of a complete and consistent normative system (completeness and consistency). However there is an important difference. Given that norms are the result of human acts of will, they may contain inconsistent demands, or may contain gaps, i.e. the normative status (obligatory, permitted, forbidden) of some regulated action is left undecided in some relevant case.

The logic of normative propositions helps to identify such defects, but this is all that legal knowledge may do. It is clear that in legal practice the interpreter goes beyond that and does eliminate contradictions or integrate gaps in order to obtain adequate justifications to individual norms. Nevertheless, this does not mean that the normative system is necessarily complete and consistent; it just means that the original system was evaluated and changed by the legal interpreter.

Here lies an important boundary between the cognitive/descriptive *versus* evaluative/prescriptive activity. Changes to restore consistency and integrate gaps are part of the prescriptive discourse (the interpreter makes choices) and a proper analysis of the legal practice (distinguishing legal cognition from legal evaluation) should make this point explicit. In (Alchourrón, 1982), when the logic of revision was in its birth, Alchourrón emphasized, as its main philosophical implication, that such model makes explicit the choice by the interpreter between possible results of a derogation (the indeterminacy problem) which is guided by a hierarchical ordering concealed in the activity of integration of law:

“In legal practice some of the cases of indeterminacy are often included in the rather ambiguous category of legal gaps. On other occasions the whole problem remains unnoticed as a consequence of what is called “a reasonable interpretation” of the legal material, concealing in this way the nature of the process of extending some of the hierarchical relations. We do not deny that such an extension might be quite reasonable; what the analysis shows is the *modus operandi* of one of the so called process of integration of law (*Rechtzergänzung*). The conceptual structure outlined in this paper makes explicit the elements involved in this very common form of legal reasoning.” (Alchourrón, 1982, p. 62)

3.1 *Logic of normative propositions*

In order to represent inconsistent and incomplete normative systems, Alchourrón's logic of normative propositions (LNP) is such that both normative gaps and contradictions are possible. LNP may be built by adding the deontic modalities O and P (respectively for obligation and pos-

itive permission) to the language of DFT. Formation rules avoid iteration of deontic modalities which are given a descriptive interpretation (Oa (Pa) means that the norm demanding (permitting) a exists in the normative system). DFT is enriched by the following axioms:

$$A1. O(a \wedge b) \leftrightarrow Oa \wedge Ob$$

$$A2. Oa \rightarrow Pa$$

$$A3. P(a \wedge b) \rightarrow Pa \wedge Pb$$

and rules of inference:

$$R1. \text{ from } a \leftrightarrow b \text{ derive } Oa \leftrightarrow Ob$$

$$R2. \text{ from } a \leftrightarrow b \text{ derive } Pa \leftrightarrow Pb$$

Note that the deontic version of the principle of non contradiction is not a theorem of the logic of normative propositions. Instead we have a definitional abbreviation for normative inconsistencies:

$$a \text{ is inconsistently regulated: } (O\neg a \wedge Pa) \leftrightarrow \text{In}(a)$$

From the definition and the axiom A2 it follows that $(O\neg a \wedge Oa) \rightarrow \text{In}(a)$. The same holds for the deontic version of the principle of excluded middle related to the possibility of normative gaps.

$$a \text{ is regulated: } (Pa \vee P\neg a) \leftrightarrow N(a)$$

Conditional norms are represented by the strict conditional with a deontic sentence in the consequent. Then we have a conditional inconsistency with respect to a in a condition b whenever $b \Rightarrow \text{In}(a)$. The concept of normative gap is not identified with the absence of regulation. There is a normative gap whenever an action a is regulated by the normative system but it does not provide a solution to some relevant cases. The relevant cases are those mentioned as conditions to the regulated action and its complementary (the negation of) conditions:

$$\text{there is a gap with respect to } a: {}^{11} G(a) \leftrightarrow (b \Rightarrow N(a) \wedge \neg b \Rightarrow \neg N(a))$$

¹¹ Note that whenever the condition is a conjunction of sentences, for instance $b \wedge c \Rightarrow N(a)$ then there is a gap if $\neg(b \wedge c) \Rightarrow \neg N(a)$ which is equivalent to $((\neg b \wedge c) \vee (b \wedge \neg c) \vee (\neg b \wedge \neg c)) \Rightarrow \neg N(a)$, that is if in any of such complementary combinations of the literals of the conjuncts the action is not regulated.

3.2 Explicit inconsistencies

Applying LNP to represent the murder example, we have the normative system S_1 given by the LNP-consequences of the following normative propositions:

- N1. $k \Rightarrow Op$ (killers ought to be punished according to system S_1)
 N2. $m \Rightarrow O\neg p$ (minors ought not to be punished according to system S_1)

From N1 and N2 one may derive $(k \wedge m) \Rightarrow (Op \wedge O\neg p)$ and hence $(k \wedge m) \Rightarrow \text{In}(p)$. This is just a case of inconsistency which may be solved by revision of the antecedent of these norms. It is a matter of decision whether N1 or N2 will be revised and such decision should not be concealed given that it corresponds to the prescriptive, not descriptive, discourse. Suppose that John, a 10 year old boy kills his parents. If the interpreter wants to consistently justify a normative solution that absolves John, the original system must be changed based on a preference for N2 over N1 for the case of minor killers. Then we have the following representation in LNP related to a new revised system S_2 :

- N1. $fk \Rightarrow Op$ (killers which are not minors ought to be punished according to system S_2)
 N2. $fm \Rightarrow O\neg p$ (minors ought not to be punished according to system S_2)

where $fk \Leftrightarrow (k \wedge \neg m)$ and $fm \Leftrightarrow m$.

Within this representation, the *creative* element of interpretation, the integration of law, becomes explicit within the modification of the knowledge set. By this move it is possible to construct legal epistemology with a clear conceptual purification from axiological considerations. It is also clear by the dynamics of change from S_1 to S_2 that the description of the normative system includes prescriptive interventions of the interpreter. Of course, *identification of meaning* and *integration of law* are intertwined: the identification of an inconsistency depends on the attribution of conflicting meanings to norm formulations and also the attribution of meaning may be restricted by the perspective of introducing a normative conflict.

The identification of meaning is taken by Alchourrón and Bulygin as a matter of investigation on the linguistic conventions of a community. Given that positivism presupposes the existence of a social convention about what the authoritative sources of law are it is an empirical task to

identify and describe what valid law is. Indeterminacy of meaning could be a problem for an objective description of the legal system. However, the analysis developed by Alchourrón and Bulygin refers to the systematization of interpreted norms, which means that it is possible to abstract from attributions of meaning. Hence, at least conceptually it is possible to distinguish both activities (systematization and interpretation) and there is no problem in accepting that there may be as many normative systems as may be offered by alternative attributions of meaning (Bulygin, 1986). Once the meaning is fixed by the analysis and a system is indexed by a particular interpretation it is possible to distinguish between the *cognitive* and *creative* steps in the description or reconstruction of what law is. Hence we may even refer to a legal system as the system which is reconstructed by the interpreter in order to justify a particular solution to an individual case.

3.3 Implicit inconsistencies

In the discussion above we have dealt with the case of explicit inconsistency within the normative system. The defeasible nature of the derived solution depended on a revision of the normative premises. As we have seen this was the first sort of legal defeasibility faced by Alchourrón.

But there is another case where a normative solution derived from the general norms is defeated because it fails to satisfy some ideal of justice or equity. In most cases this incoherence will not appear, given that normally the legislator and the interpreter share a common set of basic values. But there may be some hard cases where this is not so. The analysis of these hard cases was the starting point of Dworkin's famous attack on legal positivism (Dworkin, 1977). According to Dworkin, judges usually consider such defeating moral and political standards as binding even though they are extra-legal material. So, he advances, they are part of what law is and not of what law should be. He also emphasizes that such standards have the nature of *prima facie* duties (contributory conditionals) and their application does not follow the all-or-nothing logic (material or strict conditionals) of legal norms. In this sense such attack is directly related to the understanding of normative defeasibility.

Two main positivist responses to such attack have been proposed: one (inclusivist positivism) denies that such standards are extra-legal; the other (exclusivist positivism) denies that because they are taken as binding such standards must be part of what law is (Coleman, 2001). I believe Alchourrón and Bulygin are affiliated with the second line of defense.

Alchourrón believed that the defeasibility of legal norms by moral and political standards would be nothing but a sort of normative contradiction, this time between the explicit legal norm and the norm which is taken by the interpreter to be implicit under the legislator's intention. Suppose that John kills in self defense. According to system S_2 there is no provision for this hypothesis. Hence, given the validity of *strengthening the antecedent* for the strict conditional of LNP, it is a consequence of S_2 that killers who act in self defense ought to be punished, which is the justified normative solution to John's case. The interpreter may consider that this solution offends the moral right of every individual to defend one's life. If this is the intended justified solution to John's case this means that the following norm would be binding:

N3. Killers who acted in self defense should not be punished

On account of such cases Alchourrón and Bulygin make a distinction between the thesis of relevance of the system given by the relevant cases in the description of the system, and the hypothesis of relevance given by the cases which the interpreter believes that ought to be relevant for the system. Therefore, the hypothesis of relevance expresses a judgment about the axiological adequacy of the system (Alchourrón and Bulygin, 1971, pp. 156-157). If a condition belonging to the hypothesis of relevance is not present in the system's thesis of relevance, they speak of an *axiological gap*, which is part of the prescriptive, not the descriptive discourse. Bulygin criticizes Dworkin's attack precisely in his failure to distinguish the descriptive from the prescriptive discourse (Bulygin, 1991). That is, the principle may be binding on the interpreter and used prescriptively, but it is not part of a description of what law is.

Jorge Rodriguez claims that the hypothesis of relevance and axiological gaps should not be immediately identified with the prescriptive discourse as they may be understood as conveying information about a second normative system which is actually a reconstruction of the so called *mens legislatoris* by the legal interpreter (Rodriguez, 2000). This account gives expression to a well known argument employed in claims related to axiological gaps according to which the legislator would have provided a different solution had he considered the unpredicted circumstance.

I believe that Rodriguez's account may be captured in the dynamics of the normative system considered by the recalcitrant interpreter who changes the original set of norms into another including further qualifications. Taking our previous example in consideration we add N3 to S_2 getting S_3 :

N1. $(k \wedge \neg m) \Rightarrow Op$

N2. $m \Rightarrow O\neg p$

N3. $k \wedge d \Rightarrow O\neg p$

from which we derive $(k \wedge \neg m \wedge d) \Rightarrow In(p)$. The interpreter now integrates the system. If he intends to justify that John ought not to be punished, then in his weighing and balancing of normative reasons he should give preference to N3 over N1, which means that he will change the normative system by qualifying the antecedent of N1. Such qualification makes explicit the implicit assumption of the antecedent that the killer did not act in self-defense. The result is S_4 :

N1. $f(k \wedge \neg m) \Rightarrow Op$

N2. $fm \Rightarrow O\neg p$

N3. $f(k \wedge d) \Rightarrow O\neg p$

where $f(k \wedge \neg m) \Leftrightarrow (k \wedge \neg m \wedge \neg d)$; $fm \Leftrightarrow m$; and $f(k \wedge d) \Leftrightarrow (k \wedge d)$.

3.4 Conditionals are sufficient, as long as they last...

Alchourrón's analysis of defeasibility is centered in the process of qualification of the antecedent of general norms. Focusing on a change in an epistemic state about the law (or the world) he keeps the whole inferential power of the strict conditional. Explicit conditions are sufficient to detach consequences and through this mechanism individual normative solutions are justified and particular events are explained.

It seems that under Alchourrón's conception scientific knowledge (of the world or the law) is essentially composed by a set of general strict conditionals which explain phenomena (events or normative decisions) given the presence of antecedents. Even though he recognizes that there may be implicit assumptions and exceptions within general explanations, it is the very role of science and in particular the legal science to make them explicit (Alchourrón, 1996b, p. 342).

Hence, there seems to be no place in the scientific discourse for general laws like "water 'normally' turns into steam if the temperature is 100° C" or "according to Brazilian law if you kill you 'normally' go to jail". These defaults may be part of everyday common sense reasoning where conclusions are drawn even if there is uncertainty about actual facts.

However, in the scientific domain the overall concern regards the soundness of generalizations. Actual facts or events are only taken into account as instantiations of general hypothesis. If a generalization fails in a particular experience, e.g. "John boiled water in La Paz at a temper-

ature below 100° C”, then it must be revised by another more qualified generalization, e.g. “water turns into steam at 100° C if heated at sea level”, which is able to explain the failure as well as future events. Now, if after such refinement of the scientific theory we are not sure about pressure conditions, and are willing to infer that the water, after being subject to a particular heating, will turn into steam at 100°C, as “normally”, this is a problem related to instantiation, not to the general theory.

The same holds for legal science in the reconstruction of the legal system. If, according to the legal system, killers ought to be punished, but under a particular decision John is not punished even though he killed, such decision cannot be justified by the normative system as it stands. Certainly this decision could not be justified on the argument that the killer was John. The justification must be based on some property or circumstance of the case which could be generalized, i.e. the same normative solution would be applied to any killer in the same circumstance.¹² For instance, the decision may be justified by the fact that John was attacked by the victim, together with a general rule according to which anyone who acts in self defense cannot be criminally liable. If this is the justification, the general rule may only be accommodated through a reinterpretation which necessarily changes the legal system. The normative proposition of legal science would then associate a general solution to a refined general hypothesis like “according to the Brazilian legal system the killer who does not act in self defense ought to be punished”. Again, if Peter killed and the judge does not have enough information about the circumstances in which the action occurred, or if Peter’s lawyer does not advance a claim of self defense, then she may presume that this was not the case and “jump to the conclusion” that Peter should be punished.

Note that there is a difference when we consider defeasibility focusing the systematization of general norms/conditionals and when we consider it focusing the instantiation of a general rule. Jumping to conclusions or opening exceptions to the general rule is a matter of instantiation, whether we consider that the implicit assumptions related to the antecedent hold or do not hold in a particular case. On its turn, systematization may be provoked by failure in a particular instantiation, but the problem is how to explain such failure, which may be done by generalizing a relevant property of the exceptional case and qualifying the defeated general conditional. One may say that in the first case defeasibility has

¹² This is due to the moral principle of universalization which is entrenched in a widespread legal ideology according to which every decision should be based on general norms (see the discussion related to this principle in Alchourrón, 1996b).

to do with an *exception* in a particular instantiation while in the second case defeasibility has to do with the *qualification* of explanatory conditionals. Actually, we are dealing with the same phenomenon but seen through different glasses. Since, according to Alchourrón, the object of science is to provide general and systematized explanations of events or general and systematized normative propositions describing general norms that justify particular decisions, it is the defeasibility as *qualification* that is at stake to him.

Does that mean that under Alchourrón's account (legal) science should make explicit all possible assumptions or exceptions to causal laws and all possible circumstances in which a norm may generate a conflict in a particular instantiation or be considered unjust? This seems to be an illusion of certainty that is inadequate to a realistic account of (legal) knowledge.

In fact, Alchourrón agrees that it is impossible to formulate every implicit exception in the interpretation of a normative expression and that possibly all normative expressions are defeasible in this sense. In one of his last published papers he quotes Hart by saying that all legal norms should end with a clause "*unless...*" and also MacCormick to the effect that the attempt to formulate every conceivable expression would be the enemy of any kind of clarity or cognoscibility in law (Alchourrón 1996b, p. 341-342). However, agreeing to that just means agreeing with defeasibility of knowledge and does not commit anyone to a conception of knowledge as a set of default rules. I believe that, in Alchourrón's view knowledge of the world or of law may present itself as certain and complete by means of general strict conditionals *as long as such conditionals can cope with reality*. Defeasibility of knowledge lays in the instability of scientific propositions within the knowledge set, i.e. in the fact that they may be changed and further qualified given surprising observations or the need to justify a conflicting normative decision. After exposing the procedure of representation of defeasibility by means of general conditionals, Alchourrón makes the following comments:

"But after the second revision there arises the question: have we reached the right and desired set of premises? Usually we will never be sure to have obtained the desired (much less the right) set of premises, because we are never fully conscious of the infinite set of consequences that follows from any (even from very small) set of premises. For this reason one must be permanently checking the qualities of the premises and be ready to perform as many revisions as their analysis and our experience require." (Alchourrón, 1993, p. 79)

Now we have a clearer picture of Alchourrón's conviction why we should not conceal defeasibility by means of snakes.

First, he believed that the phenomenon of defeasibility in science carries an intrinsic relation with changes in epistemic states. Defeasibility of a conditional proposition means its susceptibility to qualification (change). Since logical consequence is not an epistemic notion (Alchourrón was not an intuitionist) a concealing non-monotonic consequence relation could not provide a philosophically adequate account of defeasibility.¹³ Second, he also believed that an adequate model of knowledge should consist in general propositions which could provide sufficient explanations of events or sufficient justifications of particular legal decisions, until they are subject to change. He agreed that there may be ordinary language conditional constructions where the antecedent is not a sufficient condition of the consequent, nevertheless he believed that we should not lose track of *modus ponens*. The only way he conceived to do that was in terms of the notion of contributory conditions. Sufficient conditions are there in a model of explanation, although they must be made explicit and are subject to change. Third, in particular, an adequate representation of legal knowledge should be restricted to the descriptive discourse, making clear the interventions of the interpreter's choices and evaluations (prescriptive discourse) that provoked changes in the original system.

In a positivist account, the representation of a legal inference should make it perfectly clear what parts of it are the *factual hypotheses*, which are given by *the existing system of norms* and which are *provided by the judge or the legal interpreter* in an effort to reach a unique conclusion in cases where there are conflicts in the law given the hypothesis or even where there are conflicts with the law and its underlying purposes or values. If we use the snakes, then the second and third elements become invisible and are easily confused with each other. On the other hand if we always represent the process as a revision, then we make the complete normative conditionals of the system explicit. The third component (the judge or interpreter's contribution) remains invisible, but comes to the surface when we analyze the process of revision itself, finding that component in the selection function or ordering that are made on the maximal consistent implicit assumptions of the normative conditionals of the system.

However, if the dynamics of change are concealed then the distinction of descriptive (description of the legal system) and prescriptive (the

¹³ A similar argument is used by Andre Fuhrmann favoring revision of an inconsistent epistemic state instead of a paraconsistent logic, i.e. a subclassical inference relation that handles inconsistent premises without trivialization (Fuhrmann, 1997).

contribution of the legal interpreter) activity in legal dogmatics may be dangerously obscured.

4. Refinement

As we have seen, according to Alchourrón, scientific knowledge is composed by general conditionals that explain reality. Failures to such explanations motivate changes in the knowledge set. These changes, which are located in the antecedent of these conditionals are formally represented by means of a revision operator, which makes explicit its implicit assumptions. The implicit assumption is the negation of the defeating factor. The defeating factor detaches an explanation to the event (or normative solution) which conflicts with the theory. On its turn, the revised conditional, with its antecedent expanded, explains a circumstance where the defeating fact does not hold. Let us generally call this move “refinement”, where a conditional of the original theory has its antecedent qualified by new conditions.

Note that no direct reference to the scientific theory is made in this operation of change. Actually, the reference to a theory is the one associated with the implicit assumptions of the antecedent of a conditional. Indeed, for each conditional you have an associated theory of implicit assumptions. In Alchourrón’s examples the refinement applies to an explicit conditional in the basis of the normative system itself. Although he adopts a Tarskian notion of a theory as a set of sentences including its logical consequences, Alchourrón does not discuss the refinement of an implicit conditional which is a consequence of the explicit basis of the system.

Of course, Alchourrón’s goal was to show the bridge between the revised conditional and the defeasible conditional, but in the procedure he favors as the adequate representation of defeasibility there is a dynamic of change of the very theory or normative system. Therefore I believe that a more faithful representation of this dynamic would be achieved by an operator on the scientific theory or normative system and not on each theory which is implicit in each antecedent of its conditionals. An alternative form of representation would be a function which refine the conflicting conditional by refining the theory itself instead of refining the theory by refining the conflicting conditional.¹⁴

¹⁴ There is a correspondence of this approach with Alchourrón and Bulygin’s theory of normative systems. They first define the concept of normative system and identify norms as its elements instead of a Kelsenian approach where first norms are defined and then the system is identified as the set of valid norms (Alchourrón and Bulygin, 1971).

Such operators were developed (Maranhão, 2001, 2006) intending to avoid inconsistencies by weakening (not elimination) of the conflicting sentences within a theory. An obvious way to do that is by restricting each sentence by incompatible conditions. The idea is very simple. Suppose one believes that water boils at 100°C (a), and so believes in this result whether one boils water at sea level or not ($b \rightarrow a$ and $\neg b \rightarrow a$). Once an experiment to boil water in La Paz fails ($\neg a$) one realizes that the real mistake was to believe that water boils at 100°C even if one is not at sea level ($b \rightarrow \neg a$, where b is the defeating condition). Therefore, what one has to do is to exclude this weaker mistaken belief preserving the other conditional according to which water boils at 100°C at sea level ($\neg b \rightarrow a$).

I called *internal refinement* the operator which restricts the original belief. The complementary move which consisted in adding the defeating conditional ($b \rightarrow \neg a$) to the theory was called *global refinement*.

The construction of the internal refinement operator is based on AGM contraction functions. I will restrict the analysis to any *interesting theory* where the sentence to be deleted is not a tautology and does belong to it (a formal definition of this notion is provided in Maranhão, 2006). First define a conditioning function h on the set of formulas that will select the sentence representing the defeating condition. Then, assuming a contraction function \div and a conditioning function h , define the internal refinement operator of a theory K by sentence a as a contraction by the defeating conditional, i.e. $K\#a = K \div h(a) \rightarrow a$. As it is shown in (Maranhão, 2006) it turns out that, given some suitable restrictions on the conditioning function, the internal refinement function is characterized by the following postulates:

- #1. $Cn(K\#a) \subseteq K\#a$
- #2. $a \notin K\#a$
- #3. $K\#a \subseteq K$
- #4. $K \subseteq Cn(K\#a \cap \{a\})$
- #5. if $a \leftrightarrow b \in Cn(\emptyset)$ then $K\#a = K\#b$
- #6. $\neg h(a) \rightarrow a \in K\#a$

This means that refinement functions are AGM-contraction functions satisfying the additional postulate #6 (*preservation postulate*) according to which the contracted sentence is preserved when the defeating condition is absent. This is exactly what Alchourrón's revision on the antecedent does, i.e. it reveals implicit assumptions which are the negation of the defeating factors. In addition, we have the guarantee that the contracted sentence is deleted (#2) and so is the sentence under the defeat-

ing condition (that $h(a) \rightarrow a \notin K\#a$ follows from #2 and #6). It also holds that the result is a smaller theory (#3, #1) that suffered a minimal change with respect to the refined sentence (#4). The logical form of the sentence to be internally refined is irrelevant (#5).

The function of global refinement of a theory K by a sentence a (notation $K \bullet a$) is constructed out of internal refinement with an obvious move which runs parallel to Levi's identity: $K \bullet a = Cn(K\#\neg a \cap \{h(\neg a) \rightarrow a\})$. It can be demonstrated that such operator satisfies the AGM revision postulates except for self-deducibility, which means that global refinement, although with obvious affinities, is not a revision.

I remark that the construction is very abstract and its results hold for any interesting theory in any underlying logic whose conditional "f" satisfies the following properties for any sentences a, b of the language: (i) $bfa \in Cn(a)$; and (ii) if $(bfa) \in Cn(\emptyset)$ and $(\neg bfa) \in Cn(\emptyset)$ then $a \in Cn(\emptyset)$. These properties are satisfied by S5 strict conditionals used by Alchourrón as the basis of DFT.

Given that refinement is an operator on theories there is no need for a revision operator on the antecedent of strict conditionals and, therefore, the language of the logic of normative proposition we will use is simpler. Instead of constructing the logic out of DFT we may depart from S5, adding the deontic modalities and their axioms as well as relevant definitions of consistency and gap. Let us call the resulting logic LNP*. Now, to illustrate Alchourrón's procedure of revision of general conditions we just refine the theory that consists in the logical consequences of S_1 :

$$(k \wedge m) \Rightarrow \text{In}(p) \in Cn(\{k \Rightarrow Op, m \Rightarrow O\neg p\}) = Cn(S_1)$$

This normative inconsistency¹⁵ is the trigger to the refinement of $Cn(S_1)$. There is a choice according to an assumed preference relation on normative propositions of the norm which is going to be refined. In the example herein N2 is preferred. Therefore, given $h(k \Rightarrow Op) = m$ we have:

$$\neg m \Rightarrow (k \Rightarrow Op) \in Cn(S_1) \# (k \Rightarrow Op) \quad (\text{preservation postulate})$$

And thus, provided that we are working with S5 strict conditional, we have:

¹⁵ Note that the presence of a *normative inconsistency* does not mean that the theory of normative propositions itself is inconsistent.

$$(k \wedge \neg m) \Rightarrow Op \in Cn(S_1) \# (k \Rightarrow Op)$$

It also holds that $(k \wedge m) \Rightarrow Op \notin Cn(S_1) \# (k \Rightarrow Op)$ and $m \Rightarrow O\neg p \in Cn(S_1) \# (k \Rightarrow Op)$ as wished to solve the inconsistency problem.

Note that by strengthening the antecedent $(m \wedge k) \Rightarrow O\neg p \in Cn(S_1) \# (k \Rightarrow Op)$ what means that global refinement is redundant in a case of explicit inconsistency. Nevertheless, if we want to refine a normative proposition by an “implicit” new circumstance, then we could use global refinement. The difference between global refinement and the procedure described in Section 3.3 is that first the defeating conditional was added obtaining S_3 and then it followed the same step as in the case of explicit inconsistency, resulting in S_4 . Global refinement provides a direct step from S_2 to S_4 (actually, if we observe the construction strictly, the defeating conditional is added after internal refinement).

A general procedure to avoid inconsistency with all the defeaters of a conditional within the theory is obtained by an internal refinement on a theory Th based on a proper selection by the conditioning function so that $h(b \Rightarrow a) = \vee \{d : d \Rightarrow \neg a \in Th\}$. The internal refinement function will then provide as output that $(b \wedge \{\neg d\}) \Rightarrow a \in Th \# (b \Rightarrow a)$.

The advantage of using an operator on the theory and not on the (antecedent of a) conditional is that we have an account of the whole resulting theory after the revision procedure. I believe that it also represents more faithfully the idea that an epistemic change in the scientific theory as a whole is performed in order to preserve its normative consistency.

Nevertheless the similarity of both approaches is evident and it is not difficult to imagine possible translations. First I note that it is possible to define a non-monotonic consequence relation or contributory conditionals using refinement to obtain the consistent set of assumptions. As we have seen $\hat{f}a$ may be seen as a revision of a theory K of implicit assumptions K^*a and the contributory conditional may be taken as $(\wedge K^*a) \Rightarrow b$.¹⁶ A parallel construction using global refinement would be $(\wedge K \bullet a) \Rightarrow b$, but, as we have seen, such conditional does not satisfy self-deductibility, which is undesirable. Alternatively, it is possible to depart from a contraction $(a \wedge K \div \neg a) \Rightarrow b$, which is equivalent to $(\wedge K^*a) \Rightarrow b$, and use internal refinement $(a \wedge K \# \neg a) \Rightarrow b$.

The key to translations between Alchourrón’s contributory conditions and refinement is the conditioning function h . Contributory conditionals may be seen as the result of refinement operators where the

¹⁶ Obviously by $\wedge K^*a$ we mean the conjunction of the elements of a finite axiomatization of K^*a .

conditioning function chooses the consistent implicit assumptions associated with the antecedent of the conditional, that is, the function h would satisfy the following restriction $h(a \Rightarrow b) = \neg(\wedge K \vdash \neg a)$. On its turn, the consistent set of implicit assumptions associated with the antecedent of a conditional $a \Rightarrow b$ may be constructed so as to observe the conditioning function of a refinement operator. One possibility would be to use a selection function s in the construction of the contraction operator on the set K of implicit assumptions satisfying the following restriction: $s(K \perp \wedge a) = \{X \in K \mid \neg a, \neg h(a \Rightarrow b) \in X\}$.

However, a full comparison between the refinement operator and Alchourrón's theory of contributory conditionals is out of the scope of the present paper and is left for future investigations.

5 Final considerations

Alchourrón's convictions about the role of science, in particular the science of the law as descriptive of general norms, led him to view defeasibility as susceptibility to change. Therefore the formal approach he considered philosophically sound to defeasibility regarded the qualification of general conditionals by means of a revision operator. Since he was also convinced that within such formal representation nothing is gained and nothing is lost in relation to the use of non-monotonic consequence relations, then, he argued, we should take the philosophically sound approach of defeasibility as qualification.

Peczenik has argued instead that the non-monotonic logics of defeasible argumentation are philosophically sound to represent legal reasoning (Peczenik, 1996). His point is that such logics constitute an adequate representation of the dialectics of advancing arguments for and against a claim, the weighing and balancing of reasons, as well as decisions which jump to conclusions based on general norms and presumptions about the facts of the case. Some also believe that such model is improper to give an account of the allocation of the *onus of proof* which is considered essential to the defeasible nature of legal argumentation.

My impression about this debate is that none of the contenders are right or that there should actually be no conflict at all. Maybe the logics of defeasible argumentation better illustrate a process which is focused on reasoning about the instantiation of general norms to particular facts, where the *onus of proof* organizes the arguments for and against a certain claim and allows us to jump to a conclusion (if a claim is not challenged by a counter-argument). This is one side of the coin in legal reasoning. But whatever is the result of such argumentation a certain nor-

mative solution is justified given some facts which were considered relevant in this process. If new facts are considered relevant so as to open an exception to the general norm present in the normative system this means that the decision can only be justified if a certain change occurs in the normative system. Such reasoning about the underlying justificatory system is the other side of the coin of legal reasoning, which perhaps better represents the logics of epistemic change like Alchourrón's logic of contributory conditionals or the logic of refinement.

Hence the contenders may be just focusing on different aspects of the same phenomenon, i.e. legal defeasibility as exception on the one hand and legal defeasibility as qualification, on the other. However, I would rather leave a deeper analysis of this dispute to another paper.

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